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Phenomenological models of elastic nucleon scattering and predictions for LHC

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Abstract

The hitherto analyses of elastic collisions of charged nucleons involving common influence of Coulomb and hadronic scattering have been based practically on West and Yennie formula. However, this approach has been shown recently to be inadequate from experimental as well as theoretical points of view. The eikonal model enabling to determine physical characteristics in impact parameter space seems to be more pertinent. The contemporary phenomenological models admit, of course, different distributions of collision processes in the impact parameter space and cannot give any definite answer. Nevertheless, some predictions for the planned LHC energy that have been given on their basis may be useful, as well as the possibility of determining the luminosity from elastic scattering.

1. Introduction

The measurements of elastic scattering of charged nucleons at present high energies [1–3] have attained ample statistics enabling to perform very precise analyses of data measured in broad interval of the four momentum transfer squared t. The region of t's where the differential cross section $\frac{d\sigma}{dt}$ can be determined covers not only the region where nearly the pure hadron (nuclear) scattering is dominant, i.e., $|t| \gtrsim 10^{-2}$ GeV^2, but also the region where the Coulomb scattering

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plays an important role, i.e., $|t| \lesssim 10^{-2}$ GeV^2 (the latter region being sometimes subdivided into Coulomb and interference parts). The complete scattering amplitude $F^{C+N}(s,t)$ defining the differential cross section fulfills (in the corresponding normalization) the relation

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2}\left|F^{C+N}(s,t)\right|^2; \tag{1.1}$$

the given amplitude being commonly decomposed according to Bethe [4] into the sum of the Coulomb scattering amplitude $F^C(s,t)$ known from QED and the hadronic amplitude $F^N(s,t)$ bound mutually with the help of relative phase $\alpha\Psi(s,t)$:

$$F^{C+N}(s,t) = e^{i\alpha\Psi(s,t)}F^C(s,t) + F^N(s,t); \tag{1.2}$$

s is the square of the center-of-mass energy, p is the momentum of an incident nucleon in the same system and $\alpha = 1/137.036$ is the fine structure constant.

The complete elastic scattering amplitude $F^{C+N}(s,t)$ used in the past has been established by Locher [5] and independently by West and Yennie [6] and has been approximately written [7,8] as

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t} f_1(t) f_2(t) e^{i\alpha\Psi(s,t)} + \frac{\sigma_{tot}(s)}{4\pi} p\sqrt{s}\big(\rho(s)+i\big)e^{B(s)t/2}. \tag{1.3}$$

The first term corresponds to the Coulomb scattering amplitude while the second term represents the elastic hadronic amplitude. The upper (lower) sign corresponds to the scattering of particles with the same (opposite) charges. The two form factors $f_1(t)$ and $f_2(t)$ in Eq. (1.3) describe the electromagnetic structure of each nucleon (commonly in a dipole form) as

$$f_j(t) = \left(1 + \frac{|t|}{0.71\ \text{GeV}^2}\right)^{-2}. \tag{1.4}$$

The relative phase $\alpha\Psi(s,t)$ in Eq. (1.3) has been shown [5,6] to be

$$\alpha\Psi(s,t) = \mp\alpha\big(\ln\big(-B(s)t/2\big) + \gamma\big) \tag{1.5}$$

where $\gamma = 0.577215$ is the Euler constant. The two quantities, i.e., the diffractive slope B and the ρ (the ratio of the real to imaginary parts of hadronic component in forward direction) specifying the hadronic amplitude are t independent; together with the total cross section σ_{tot} they can depend only on the energy. The influence of spins of all particles involved in the elastic scattering was neglected when simplified West and Yennie complete amplitude (1.3) was derived.

Eq. (1.5) has been derived provided some assumptions concerning the t dependence of the hadronic elastic amplitude (the second term on its right-hand side) in Eq. (1.3) and also some high-energy approximations have been used. They were acceptable in the time when formulas (1.3) and (1.5) were derived and when nothing was known about diffraction structure of elastic scattering data. However, the questions have arisen when experimental data have shown not to be in agreement with corresponding assumptions.

The presented paper consists then in principle of two parts. The first part deals with the description of common influence of both the Coulomb and elastic hadronic interactions at high energy elastic nucleon collisions. In Section 2 we shall show in more detail the assumptions on which the formulas (1.3) and (1.5) are based and what are the limits of their use in analyses of contemporary experimental data. It will be shown that neither the assumptions nor the theoretical approaches used in describing the mentioned processes are fully theoretically consistent presently. In Section 3 we shall discuss the alternative approach based on the eikonal model which not only removes the corresponding limitations but which allows also to describe the common effect of both the Coulomb and hadronic interactions in the whole measured region of momentum transfers with the help of only one formula. Its detailed analytical form will be given, being convenient for the application to analysis of experimental data or for obtaining the predictions of phenomenological models including the influence of Coulomb scattering at high values of $|t|$. The t dependence of elastic hadronic scattering amplitude $F^N(s,t)$ derived from experimental data within the eikonal model enables then to determine some interesting physical characteristics in the framework of impact parameter space. They are the mean values of impact parameter corresponding to different kinds of scattering processes; corresponding formulas being presented.

In the second part the eikonal model has been made use of to derive some predictions concerning the LHC experiments. In Section 4 the assumptions used in the derivation of simplified West and Yennie formula (Eqs. (1.3) and (1.5)) will be analyzed on the basis of complete eikonal amplitude. The approach will be applied to pp elastic scattering data at the energy of 53 GeV. In Section 5 the general eikonal model approach will be used in deriving the predictions of several phenomenological models proposed for the description of elastic pp scattering at nominal LHC energy of 14 TeV. The problems connected with the estimation of luminosity on the basis of elastic nucleon scattering will be analyzed in Section 6. The calculated root-mean-square (RMS) values of total, elastic and inelastic impact parameters corresponding to individual analyzed models will be then given and discussed in Section 7. And the results obtained on the basis of our approach will be summarized in Section 8.

2. The West and Yennie formula

The general shape of phase function $\Psi(s,t)$ entering into Eq. (1.2) has been derived by West and Yennie [6] within the framework of Feynman diagram technique (one photon exchange) in the case of charged point-like particles and for $s \gg m^2$ (m standing for nucleon mass) as

$$\Psi_{WY}(s,t) = \mp\left(\ln\frac{-s}{t} - \int_{-4p^2}^{0}\frac{dt'}{|t'-t|}\left[1-\frac{F^N(s,t')}{F^N(s,t)}\right]\right), \tag{2.1}$$

which was further simplified and Eqs. (1.3) and (1.5) were derived.

The simplification was based on some assumptions concerning both the modulus and the phase of the elastic hadronic amplitude defined as

$$F^N(s,t) = i\left|F^N(s,t)\right|e^{-i\zeta^N(s,t)}. \tag{2.2}$$

As nothing was known about the experimental behavior of $\frac{d\sigma}{dt}$ at higher $|t|$ at that time the following crucial assumptions were accepted:

- the t dependence of the modulus of the elastic hadronic amplitude has been taken as purely exponential for *all* kinematically allowed t values,
- both the real and imaginary parts of the elastic hadronic amplitude have been assumed to exhibit the same t dependence again for *all* kinematically allowed t values; their ratio being constant.

In addition to these assumptions, some other high energy approximations were added (for details see, e.g., Refs. [6,9–11]). And in principle good standard fits have been obtained when the given formulas have been applied to the Coulomb and interference domains. One cannot be, of course, sure about the actual meaning of the parameters obtained by fitting as they depend strongly on the assumptions concerning the behavior at all higher values of $|t|$. However, formulas (1.3)–(1.5) have been used for fitting the experimental data of differential cross section practically in all the experiments of elastic hadronic scattering in the regions of small $|t|$ without taking into account what is the t dependence of elastic scattering at higher $|t|$ values.

The three quantities σ_{tot}, B and ρ (at corresponding energy values) have been always established on the basis of Eqs. (1.1)–(1.5) from the experimental data obtained at small values of $|t|$ (in the Coulomb, interference and also in a small adjacent part of hadronic domain). As to the larger $|t|$ values (i.e., in the hadronic region) the influence of Coulomb scattering has been usually fully neglected and elastic scattering has been described with the help of phenomenological elastic hadronic amplitude $F^N(s,t)$ which usually has exhibited a much more complicated t dependence in this hadronic region than required by Eq. (1.3). The different regions of differential cross section have been described, therefore, by *two different formulas* (based on incompatible assumptions), which is to be denoted as important deficiency.

In some papers (see, e.g., Refs. [12,13]) the complete scattering amplitude $F^{C+N}(s,t)$ has been, therefore, described with the help of one approximate formula, i.e., with the help of Eq. (1.3) containing the standard West and Yennie (WY) phase (1.5) and the elastic hadronic amplitude $F^N(s,t)$ (substituting the second term in Eq. (1.3)) constructed on the basis of some phenomenological ideas deviating from the two assumptions under which Eqs. (1.3) and (1.5) have been derived.

It might seem that a correct way may consist in reverting back to integral formula (2.1). However, that is not possible, either, if the phase $\Psi_{WY}(s,t)$ should be real for any t (see Eq. (1.2)) as generally assumed [4]. In this case the imaginary part of Eq. (2.1) should equal identically to zero, i.e.,

$$\Im\big[\Psi_{WY}(s,t)\big] = -\int_{-4p^2}^{0} dt' \, \frac{\sin[\zeta^N(s,t) - \zeta^N(s,t')]}{|t'-t|} \frac{|F^N(s,t')|}{|F^N(s,t)|} \equiv 0. \tag{2.3}$$

It means that the following condition should equal zero identically for each kinematically allowed value of t:

$$\int_{-4p^2}^{t} dt' \, \frac{\sin[\zeta^N(s,t) - \zeta^N(s,t')]}{t-t'} \big|F^N\big(s,t'\big)\big| - \int_{t}^{0} dt' \, \frac{\sin[\zeta^N(s,t) - \zeta^N(s,t')]}{t-t'} \big|F^N\big(s,t'\big)\big| \equiv 0. \tag{2.4}$$

This condition represents the nonlinear singular Cauchy type integral equation of the first kind of non-continuous function with parameter. Evidently it has a zero solution, i.e.,

$$\zeta(s,t) - \zeta\big(s,t'\big) \equiv 0 \quad \Rightarrow \quad \zeta(s,t) \equiv \zeta(s). \tag{2.5}$$

A question arises if this solution is unique or not. This problem has been solved in Ref. [14] where it has been proved analytically that there is only one solution of Eq. (2.4), i.e., that the hadronic phase $\zeta(s,t) \equiv \zeta(s)$ is t independent for the phase $\zeta(s,t)$ limited by the condition

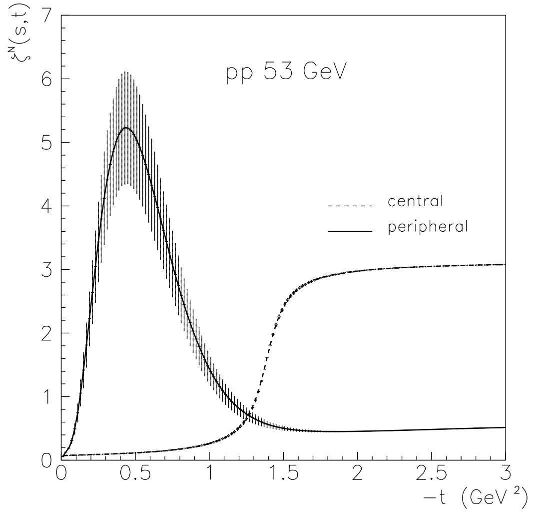

Fig. 1. Two different t dependences of hadronic phase $\zeta^N(s,t)$; one leading to the central picture of elastic hadronic pp scattering at 53 GeV (dashed line) and the second (full line) giving the peripheral picture.

$$\left|\zeta(s,t)\right| < \pi. \tag{2.6}$$

It means that *the relative phase factor $\Psi_{WY}(s,t)$ may be real only provided the phase of the elastic hadronic amplitude $\zeta^N(s,t)$ is t independent in the whole region of kinematically allowed t values* [14]; i.e., the quantity $\rho(s,t)$ should be constant in the whole interval of t. It means that if the quantity ρ is t dependent then the phase factor $\Psi_{WY}(s,t)$ is a complex quantity.

This can be confirmed also numerically by evaluating the imaginary part of WY integral (2.1) (the left-hand side of Eq. (2.3)) for any chosen t dependent quantity $\rho(s,t)$, or equivalently for any t dependent elastic hadronic phase $\zeta^N(s,t)$. Fig. 1 shows the t dependence of two different elastic hadronic phases $\zeta^N(s,t)$ derived alternatively for the elastic pp scattering at energy of 53 GeV (see Ref. [15]). The first one changes significantly (goes to $\pi/2$) only around $-t \sim 1.4$ GeV^2 (the phase $\zeta^N(s,t)$ is limited by the condition $|\zeta^N(s,t)| < \pi$) and leads to a standard 'central' picture of elastic hadronic scattering in the impact parameter space. The corresponding t dependences of the imaginary part of relative WY phase $\alpha\Psi_{WY}$ is shown in Fig. 2; at small $|t|$ values the imaginary part is different from zero and the WY relative phase becomes complex. And the second 'peripheral' t dependence of the elastic hadronic phase $\zeta^N(s,t)$ is limited by broader condition $|\zeta^N(s,t)| < 2\pi$ in Fig. 1, giving the 'peripheral' picture of elastic hadronic scattering in the impact parameter space; it corresponds to the second graph in Fig. 2. The imaginary part of the relative phase $\alpha\Psi_{WY}$ oscillates around zero at smaller $|t|$ values. Also in this case the WY relative phase is complex.

The phenomenological models of high energy elastic nucleon scattering applied to the contemporary experimental data show, however, convincingly that the quantity ρ cannot be t independent. Therefore, one should conclude that also the integral formula (2.1) should be designated as inadequate for the description of elastic hadronic scattering.

In spite of this fact some people have tried to extend the validity of simplified WY complete amplitude to much broader region of t for any t dependence of elastic hadronic amplitude $F^N(s,t)$ (i.e., for any t dependence of its modulus $|F^N(s,t)|$ and also of its phase $\zeta^N(s,t)$) by introducing some corrections to the basic WY phase (1.5). This has been done, e.g., in Ref. [16]

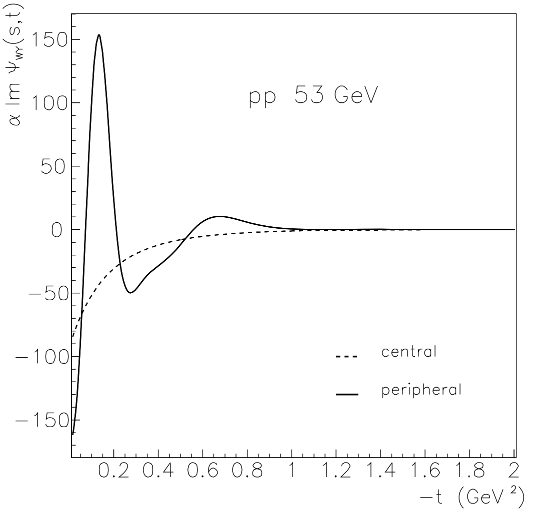

Fig. 2. The non zero imaginary parts of WY relative phase $\alpha\Psi_{WY}$ corresponding to the hadronic phases from Fig. 1 – calculated numerically.

where the corrections corresponding to the Gaussian shape of electromagnetic form factor and of elastic hadronic amplitude have been admitted. Thus practically only the deviations from the purely exponential t dependence of the modulus have been considered. However, prospective t dependence of the quantity $\rho(s,t)$ has not been taken into account in this paper.

Another approach has been chosen in Ref. [17]. Here two assumptions looking as very natural but implicitly very limiting have been used. First, the used formulation of impact parameter representation of the scattering amplitude has not been valid at finite energies but only at infinite energies. Second, the used t dependence of the hadronic form factor (dipole) representing spatial charge distribution of nucleons has been expanded into three terms leading to the Born expansion of the relative phase up to the second order and the corrections to the WY relative phase with the help of the expression containing in general complex hadronic amplitude have been calculated (for detail see Eq. (41) and Fig. 2 of Ref. [17]). As the result the calculated corrections to the real relative WY phase are complex. Thus the relative phase becomes complex and the phase looses its physical sense.

Both these approaches may be, therefore, hardly used as convenient tool for a consistent description of common influence of both the Coulomb and hadronic scattering exhibiting a general t dependence of both the modulus and phase of elastic hadronic amplitude. It is necessary to give decisive preference to a new and more suitable approach based on eikonal model. In the following we should like to demonstrate the possibilities and advantages of the eikonal model which is more general and more appropriate than that of West and Yennie.

3. Eikonal model approach and mean values of impact parameter

It has been shown in the papers of Adachi et al. [18] that the complete elastic scattering amplitude $F^{C+N}(s,t)$ may be related to the complete elastic scattering eikonal $\delta^{C+N}(s,b)$ by Fourier–Bessel transformation

$$F^{C+N}\big(s,q^2=-t\big)=\frac{s}{4\pi i}\int\limits_{\Omega_b} d^2b\, e^{i\vec{q}\vec{b}}\big[e^{2i\delta^{C+N}(s,b)}-1\big], \tag{3.1}$$

where Ω_b is the two-dimensional Euclidean space of the impact parameter $\vec{b}$. If this formula is to be applied to finite energies some problems appear as the amplitude $F^{C+N}(s,t)$ is defined in a finite region of t only. Mathematically consistent use of Fourier–Bessel transformation requires the existence of the reverse transformation, too. And it is necessary to take into account the values of elastic amplitude from unphysical region where the elastic hadronic amplitude is not defined; for details see Refs. [18]. This issue has been resolved in a unique way by Islam [19,20] by analytically continuing the elastic hadronic amplitude $F^N(s,t)$ from the physical to the unphysical region of t. Then the elastic hadronic amplitude in the impact parameter space consists of two terms

$$\begin{aligned} h_{el}(s,b) &= h_1(s,b)+h_2(s,b)\\ &= \int\limits_{t_{\min}}^{0} dt\, F^N(s,t)J_0\big(b\sqrt{-t}\big)+\int\limits_{-\infty}^{t_{\min}} dt\, F^N(s,t)J_0\big(b\sqrt{-t}\big); \end{aligned} \tag{3.2}$$

and a similar relation holds also for the representation of inelastic overlap function $g_{inel}(s,b)$ [21] in the impact parameter space. Then the unitarity equation in the impact parameter space can be approximated (see Refs. [18] and [19]) as

$$\Im h_1(s,b)=\big|h_1(s,b)\big|^2+g_1(s,b). \tag{3.3}$$

And the total cross section, integrated elastic and inelastic cross sections may be obtained also as

$$\begin{aligned} &\sigma_{tot}(s)=8\pi\int\limits_0^\infty b\,db\,\Im h_1(s,b),\\ &\sigma_{el}(s)=8\pi\int\limits_0^\infty b\,db\,\big|h_1(s,b)\big|^2,\qquad \sigma_{inel}(s)=8\pi\int\limits_0^\infty b\,db\,g_1(s,b). \end{aligned} \tag{3.4}$$

The functions $\Im h_1(s,b)$ and $|h_1(s,b)|^2$ represent then two main impact parameter profiles (for total and elastic processes) and describe the intensity of interactions between two colliding particles in the dependence on their mutual impact parameter.

The complete elastic eikonal $\delta^{C+N}(s,b)$ may be expressed as the sum of both the Coulomb $\delta^C(s,b)$ and hadronic $\delta^N(s,b)$ eikonals at the same value of impact parameter b [22]:

$$\delta^{C+N}(s,b)=\delta^C(s,b)+\delta^N(s,b) \tag{3.5}$$

and the individual eikonals may be defined as integrals of corresponding potentials [23].

The complete elastic scattering amplitude may be then written as [22–24,15]

$$F^{C+N}(s,t)=F^C(s,t)+F^N(s,t)+\frac{i}{\pi s}\int\limits_{\Omega_{q'}} d^2q'\,F^C\big(s,q'^2\big)F^N\big(s,\big[\vec{q}-\vec{q}'\big]^2\big). \tag{3.6}$$

This equation containing the convolution integral differs substantially from Eq. (1.2). In the final form (valid at any s and t up to the terms linear in α) it may be written [15] as

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t} f_1(t) f_2(t) + F^N(s,t)\big[1 \mp i\alpha G(s,t)\big], \tag{3.7}$$

where

$$G(s,t) = \int\limits_{-4p^2}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'}\big[f_1(t') f_2(t')\big] + \frac{1}{2\pi}\left[\frac{F^N(s,t')}{F^N(s,t)} - 1\right] I(t,t') \right\}, \tag{3.8}$$

and

$$I(t,t') = \int\limits_0^{2\pi} d\Phi'' \frac{f_1(t'') f_2(t'')}{t''}, \quad t'' = t + t' + 2\sqrt{tt'}\cos\Phi''. \tag{3.9}$$

Instead of the t independent quantities B and ρ, it is now necessary to consider corresponding t dependent quantities defined as

$$B(s,t) = \frac{d}{dt}\left[\ln\frac{d\sigma^N}{dt}\right] = \frac{2}{|F^N(s,t)|}\frac{d}{dt}\big|F^N(s,t)\big| \tag{3.10}$$

and

$$\rho(s,t) = \frac{\Re F^N(s,t)}{\Im F^N(s,t)}. \tag{3.11}$$

The total cross section derived with the help of the optical theorem is then given by

$$\sigma_{tot}(s) = \frac{4\pi}{p\sqrt{s}} \Im F^N(s, t=0). \tag{3.12}$$

The form factors $f_1(t)$ and $f_2(t)$ reflect the electromagnetic structure of colliding nucleons and form a part of the Coulomb amplitude from the very beginning. Due to the integration over all kinematically allowed region of t in Eq. (3.8) their actual t parameterization would describe the electromagnetic structure of the proton in as largest interval of t as possible. This has been the reason why instead of using the dipole form factor (1.4) as it has been done in Eq. (1.3) it has been suggested to use more convenient formula from Refs. [28,29]:

$$f_j(t) = \sum_{k=1}^{4} \frac{g_k}{w_k - t}, \quad j = 1,2 \tag{3.13}$$

where the values of the parameters g_k and w_k have been taken from Ref. [29]; their renormalized values are given in Table 1.

The form of form factors enables then analytical calculation of the integral in Eq. (3.9):

$$I(t,t') = \sum_{j,k=1}^{4} g_j g_k W_{jk} I_{jk} \tag{3.14}$$

where for $j \neq k$ it holds

$$I_{jk} = 2\pi\left[\frac{(P_j - 1)^2}{\sqrt{P_j}(P_j - P_k)(P_j - U)} + \frac{(P_k - 1)^2}{\sqrt{P_k}(P_j - P_k)(U - P_k)} + \frac{(U - 1)^2}{\sqrt{U}(U - P_j)(U - P_k)}\right]. \tag{3.15}$$

Table 1
The values of parameters specifying the four-pole form factor taken from Ref. [29]; where the parameters (originally expressed in units of fm^{-2}) are transfered for our purposes to units of GeV^2.

k	1	2	3	4
g_k	0.0301	0.8018	−1.0882	0.2642
w_k	0.1375	0.5848	1.7164	6.0042

For $j = k$ one has

$$I_{jj} = 2\pi \left[\frac{(P_j - 1)(3P_j + P_j^2 - U - 3P_j U)}{2P_j^{3/2}(P_j - U)^2} + \frac{(U-1)^2}{\sqrt{U}(U - P_j)^2} \right], \tag{3.16}$$

where

$$P_j = \frac{w_j + (\sqrt{-t} + \sqrt{-t'})^2}{w_j + (\sqrt{-t} - \sqrt{-t'})^2}, \qquad U = \left(\frac{\sqrt{-t} + \sqrt{-t'}}{\sqrt{-t} - \sqrt{-t'}} \right)^2 \tag{3.17}$$

and

$$W_{jk} = \frac{1}{[w_j + (\sqrt{-t} - \sqrt{-t'})^2][w_k + (\sqrt{-t} - \sqrt{-t'})^2][\sqrt{-t} - \sqrt{-t'}]^2}. \tag{3.18}$$

The influence of Coulomb scattering in elastic proton collisions represents a rather complicated problem. The electromagnetic proton structure is usually determined from the elastic electron proton scattering. Its differential cross section is given by Rosenbluth formula [25] (i.e., in the one photon exchange)

$$\frac{d\sigma}{d\Omega}(E_0, \theta) = \frac{d\sigma}{d\Omega}(E_0, \theta)\bigg|_{NS} \left[a(q^2) G_E^2(q^2) + b(q^2, \theta) G_M^2(q^2) \right], \tag{3.19}$$

$$a(q^2) = \frac{1}{1+\tau}, \qquad b(q^2, \theta) = \mu^2 \left(\frac{\tau}{1+\tau} + 2\tau \tan^2 \theta/2 \right),$$

$$\tau = \frac{q^2}{4m^2}, \qquad G_E(0) = G_M(0) = 1. \tag{3.20}$$

Here $\frac{d\sigma}{d\Omega}(E_0, \theta)|_{NS}$ is the elastic cross section between a Dirac electron and a point-like and spinless point charge particle of proton mass m at incident energy E_0 and scattering angle θ (in Born approximation). The formula contains the electric form factor $G_E(q^2)$ and the magnetic one $G_M(q^2)$ which have been introduced by Sachs [26]. Let us note that both the form factors depend only on the square of exchanged momentum transfer $t = -q^2$.

The t dependence of electric form factor has been earlier empirically approximately described [27] by a dipole fit according to Eq. (1.4)

$$G_D(t) \equiv f_j(t) = 1/(1 - t/0.71)^{-2}, \quad [t] = \mathrm{GeV}^2 \tag{3.21}$$

while the magnetic form factor has fulfilled scaling relation

$$G_E(t) = G_M(t)/\mu, \tag{3.22}$$

where μ is the magnetic moment of the proton. In Refs. [28,29] electron proton elastic scattering data at several energies have been analyzed with the help of Rosenbluth differential cross section formula where the t dependences of both the form factors have been parameterized according to Borkowski‘s formula (3.13) (inspired by the vector dominance model). The best four-pole fit of free parameters has shown that their values are different for electric and magnetic proton form factors (see Table 4 in Ref. [29]) and that the scaling equation (3.22) has been violated. The resulting fit has also shown that Borkowski‘s electric form factor describes the elastic *ep* differential cross section much better than the electric dipole form factor. This fact seems to be also confirmed in Refs. [30,31] and mainly in Ref. [32] (see Table II) where the deviations of ratio $G_E(t)/G_D(t)$ from 1 are significant – especially in the region of $-t \in (0.1, 5.85)$ GeV^2. Thus the use of Borkowski's parameterization (3.13) seems to be justified provided the shape of form factors derived from *ep* elastic scattering is the same as the shape of form factors involved in the description of elastic *pp* scattering.

As the Coulomb part in formula (3.7) is known the complete elastic amplitude depends in principle on hadronic amplitude $F^N(s,t)$ only. At difference to the WY approach Eq. (3.7) enables to determine the complete elastic scattering amplitude $F^{C+N}(s,t)$ for any t dependent elastic hadronic amplitude $F^N(s,t)$ in the whole measured region of t. The differences between these two approaches have been sufficiently described in Refs. [14,15]. Partially they will be illustrated in the case of *pp* elastic scattering at the energy of 53 GeV in Section 4.

Thus the given approach may be used in two complementary ways:

- one can test the predictions of different models of high-energy elastic hadronic scattering that provide hadronic amplitude $F^N(s,t)$. With the help of formula (3.7) one can calculate complete amplitudes $F^{C+N}(s,t)$ that can be compared to experimental data by employing Eq. (1.1);
- one may resolve phenomenological t dependence of elastic hadronic amplitude $F^N(s,t)$ at a given s (and for *all measured t values*), by fitting experimental elastic differential cross section data with the help of Eqs. (1.1) and (3.7). The crucial point here is then a suitable parameterization of the hadronic amplitude $F^N(s,t)$.

The eikonal approach brings then the possibility of determining mean values of impact parameter for different kinds of scattering processes. These quantities may characterize the ranges of forces responsible for the elastic, inelastic and total scattering. If the unitarity condition and the optical theorem are made use of the mean-square values of impact parameter for different processes may be determined directly from the t dependence of elastic hadronic amplitude $F^N(s,t)$.

The elastic mean-square may be determined by means of the formula (see Refs. [33–36])

$$\begin{aligned}\langle b^2(s)\rangle_{el} &= 4\frac{\int_{t_{\min}}^0 dt|t|(\frac{d}{dt}|F^N(s,t)|)^2}{\int_{t_{\min}}^0 dt|F^N(s,t)|^2} + 4\frac{\int_{t_{\min}}^0 dt|t||F^N(s,t)|^2(\frac{d}{dt}\zeta^N(s,t))^2}{\int_{t_{\min}}^0 dt|F^N(s,t)|^2}\\ &\equiv \langle b^2(s)\rangle_{mod} + \langle b^2(s)\rangle_{ph}\end{aligned} \tag{3.23}$$

where the modulus of elastic hadronic amplitude itself determines the first term and the phase (its derivative) influences the second term only; note that both the terms are positive.

The total mean-square (for all collision processes) may be determined with the help of the optical theorem by (see Ref. [35])

$$\langle b^2(s)\rangle_{tot} = 2B(s,0); \tag{3.24}$$

the diffractive slope $B(s,t)$ being defined by Eq. (3.10).

According to the unitarity equation the averaged inelastic mean-square is related to the total and elastic mean-squares as [35]

$$\langle b^2(s)\rangle_{inel} = \frac{\sigma_{tot}(s)}{\sigma_{inel}(s)}\langle b^2(s)\rangle_{tot} - \frac{\sigma_{el}(s)}{\sigma_{inel}(s)}\langle b^2(s)\rangle_{el}. \tag{3.25}$$

All the formulas given in the preceding three sections form the basis of formulas which will be used for obtaining the predictions of various phenomenological approaches studied in the following sections.

4. Eikonal model and assumptions from simplified West and Yennie approach

This section contains the discussion of the eikonal model approach, described in the previous section; the consequences of two crucial assumptions will be analyzed, under which the simplified WY formula for the complete interference elastic amplitude has been derived. For this reason the data of pp elastic scattering at the ISR energy of 53 GeV taken from Ref. [37] will be used; they describe the $\frac{d\sigma}{dt}$ in the broadest interval of measured t values running from 0 to 10 GeV^2. As the complete eikonal model amplitude defined by means of Eqs. (3.7)–(3.9) contains the integration over all kinematically allowed t values the numerical analysis will be the better, the broadest region of t will be applied to. The basic approach has been originally made use of in Ref. [15] where convenient parameterizations of both the modulus and the phase of the elastic hadronic amplitude $F^N(s,t)$ as

$$\left|F^N(s,t)\right| = (a_1 + a_2 t)e^{b_1 t + b_2 t^2 + b_3 t^3} + (c_1 + c_2 t)e^{d_1 t + d_2 t^2 + d_3 t^3} \tag{4.1}$$

and

$$\zeta^N(s,t) = \arctan\frac{\rho_0}{1 - |\frac{t}{t_{diff}}|} \tag{4.2}$$

have been used. The t dependences of both these quantities have been practically the same as in the majority of common phenomenological models and the two assumptions under which the simplified WY complete interference formula has been derived may be easily incorporated.

The complete elastic scattering amplitude limited by different assumptions may be calculated with the help of Eqs. (3.7)–(3.18) using the hadronic amplitude $F^N(s,t)$ parameterized according to Eqs. (4.1) and (4.2). The results are shown in Fig. 3 where the best fits of the differential cross sections under different assumptions have been given [38]. Four different alternatives have been studied. The contribution of the mere Coulomb interaction corresponds to the dashed and dotted line; and second, the contribution of the pure hadronic interaction and corresponding to $\rho = \text{const.}$ and $B = \text{const.}$ is represented by dotted line. When the Coulomb scattering is included to this case the graph with a dashed line is obtained. And finally, when the interference between the Coulomb and hadronic scattering with $\rho = \text{const}$ and $B(t)$ (parameterized according to Eq. (4.1)) is demonstrated by the full line roughly copying the data. Compared to the case from Ref. [15], where both the parameters $\rho(t)$ and $B(t)$ were fitted, there is only a little higher value of χ^2.

This type of analysis shows quite surely that the application of simplified WY formula for the analysis of high-energy elastic scattering data is unsuitable at the present. It might be acceptable when Locher [5] and West and Yennie [6] derived it as the diffractive structure of differential cross section was not known. However, the contemporary phenomenological models of high-energy elastic hadron collisions exhibit rather complicated t dependence which can be correctly incorporated provided the general t dependent hadronic component $F^N(s,t)$ (the quantity ρ of which is t dependent) is applied to. Such an attitude seems to be supported also by the theorem of Martin [39] saying that the real part of the elastic hadronic amplitude has to change its sign at

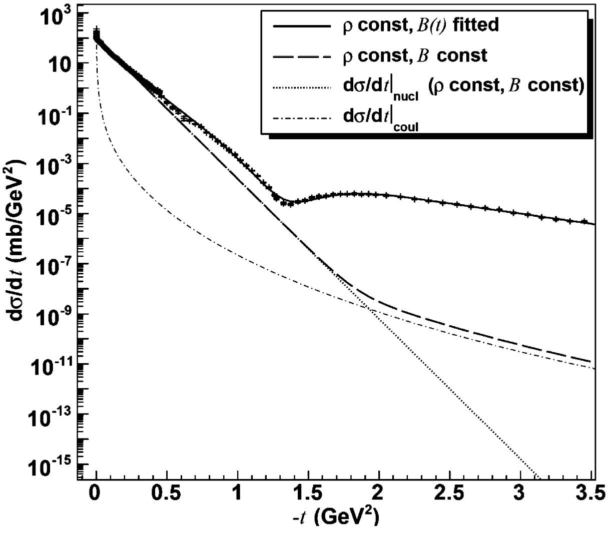

Fig. 3. Different contributions to $\frac{d\sigma}{dt}$ for pp scattering at 53 GeV.

small $|t|$ if the total cross section tends to infinity and also the differential cross section tends to zero for fixed t when s increases to infinity.

5. Models for *pp* elastic scattering and their predictions at the LHC energy

In the past many phenomenological models describing high-energy nucleon collisions have been constructed on different levels of sophistication. In many of them the simple eikonal approach has been made use of; e.g., as in the model of Bourelly, Soffer and Wu (see Refs. [40, 41]). In other models the nucleons have been assumed to be composed of a central core and of surrounding meson cloud [42–44] or of a series of parton interaction centers and have described hadron collisions with the help of Glauber's multi scattering method [45,46]. The contributions of three pomerons have been also considered in papers [47,48]. A further class of approaches has started from the nucleon structure and identified the partons with quarks and gluons; the hadron scattering has been described as a sum of pomeron or gluon exchanges [49] or as a semihard scattering of quarks and gluons [50–53]. Other approaches have tried to describe high-energy diffractive nucleon–nucleon scattering within non-perturbative QCD, using the model of stochastic vacuum [54,55].

5.1. Some common characteristics of the models

All considered models have exhibit some basic characteristics that may be summarized as it follows:

- Imaginary parts of all elastic hadronic scattering amplitudes have been taken as dominant in a rather broad region of momentum transfers around forward direction and vanishing in the region of diffractive minimum. The given dominance may be justified by asymptotic theorems but practically only in very forward direction at present energies [10].

- The real part of elastic hadronic amplitude (e.g., pp scattering) has usually decreased from its positive value at $t = 0$, become negative at small $|t|$ and filled the dip in $\frac{d\sigma}{dt}$ in the region of diffractive minimum.
- The used eikonal or impact parameter representation of the scattering amplitude has been valid only at infinite energies. Only in this case the physical region of t variable is unlimited from below, i.e., $t \in (-\infty, 0)$ as required by consistently defined Fourier–Bessel transformation. However, at finite energies when the physical region of t variable has been limited from below the mathematically correct use of FB transformation has required the scattering amplitude to be defined also in the unphysical region of t. In such a case the FB image of the scattering amplitude oscillates at higher impact parameter values – for details see Refs. [18–20].
- Common dynamical assumption: distribution of hadronic matter has been roughly the same as distribution of electric charge inside hadrons.
- The corresponding elastic hadronic amplitudes have been analytic, unitary and satisfied crossing symmetry and the Froissart–Martin bound [56–58].
- In the QCD inspired models the properties of nucleon scattering amplitudes known on the hadronic level have been simply attributed to the amplitudes on the quark level; the energy and impact parameter dependences of the eikonals have been factorized.
- The common influence of both the Coulomb and the elastic hadronic scattering has not been described in some papers correctly. The complete simplified WY elastic amplitude has been used – see Eqs. (1.3) and (1.5) – where the original WY form of elastic hadronic amplitude has been substituted by a new elastic hadronic amplitude not fulfilling the assumptions, under which the complete WY amplitude has been derived.

All mentioned models have included the factors allowing to study the energy dependence of individual differential cross sections. And as the members of TOTEM [59,60] Collaboration we have attempted to make use of them to give all standard predictions concerning the nominal LHC energy of 14 TeV. Besides the performed analyses have enabled to obtain also the total, elastic and inelastic profiles characterizing the distributions of corresponding collisions in the impact parameter space. Four contemporary models proposed by Bourrely, Soffer and Wu [40,41], Petrov, Predazzi and Prokudin [47,48], Block, Gregores, Halzen and Pancheri [51] and Islam, Luddy and Prokudin [43,44] will be discussed in the following.

The first analyzed model (Bourelly, Soffer and Wu [40,41]) has started from the eikonal form of elastic hadronic amplitude defined by Eq. (3.1) in the impact parameter space. The elastic hadronic amplitude has been generated by the complex eikonal function which is the function of s and b; these two dependences have been assumed to be factorized (with the exception of added small Regge background term corresponding to exchange of (A_2, ρ, ω) Regge trajectories). The first energy-dependent complex factor has been crossing symmetric while the other factor depending only on b has been identified with the FB transformation of the electromagnetic dipole form factor of the nucleon times some slowly varying function expressing small differences between the distribution of electric and hadronic "charges" of nucleons.

The second analyzed model of Petrov, Predazzi and Prokudin [47,48] has belonged to the standard Regge type models. Its elastic hadronic amplitude has been represented by the eikonal elastic amplitude introduced by Eq. (3.1) where the complex eikonal function has been decomposed into six terms: the first three ones have corresponded to the three pomerons (generated by the exchange of pomeron linear trajectories with decreasing slopes). The next eikonals have corresponded to the odderon contribution and to the contributions of mesons f and ω. Two alternatives of this model have been proposed by the authors: one corresponding to the exchange of two pomerons and second to three pomeron exchange.

The next analyzed model has been the QCD-inspired model of Block, Gregores, Halzen, and Pancheri [51,52]. Its elastic scattering amplitude has had the same eikonal form as it is in Eq. (3.1). Its complex eikonal function has been proposed to be decomposed into quark–quark, quark–gluon and gluon–gluon contributions. Each eikonal component has been factorizable into a product of cross section of colliding partons times normalized impact parameter overlap function parametrized as the FB transformation of the corresponding dipole form factors.

The last analyzed model has been the model of Islam, Luddy and Prokudin [43,44]. Its elastic hadronic amplitude has been represented by a sum of the three terms. The first one has been taken as the FB transformation of phenomenologically introduced diffractive component. The second one has been the hard scattering amplitude which has originated from one nucleon core scattering off the other core via vector meson ω exchange, while their outer clouds has overlapped and interacted independently. And the third contribution has come from the hard pomeron term describing the hard collision of a valence quark from one proton with a valence quark from the other proton.

However, none of the proposed models of pp elastic scattering at the LHC has not taken into account the influence of the proton spin. Partly due to the fact that the scattering of polarized protons at the LHC energies is not on the program. The elastic scattering of polarized protons at high energies has been studied theoretically in Ref. [61] but only in the region of small $|t|$ values near the forward direction. The resulting unpolarized differential cross section has been then determined by contributions of five complete helicity amplitudes each of which has been given by the sum of electromagnetic and elastic nuclear (hadronic) helicity components mutually bounded by the relative phase factor similar to the current WY form – see Eq. (1.5). Using the high energy assumptions similar to those needed for derivation of Eq. (1.5), i.e., one photon approximation of electromagnetic interaction and purely exponential t dependence of the individual hadronic helicity components it has been shown that all relative phases between the electromagnetic and hadronic helicity components are identical. However, the t dependence of each elastic hadronic helicity amplitude may be hardly specified at present at the LHC energies.

5.2. *Model predictions*

All the mentioned phenomenological models of high energy nucleon collisions have been represented by their elastic hadronic amplitudes. And in case of the scattering of charged nucleons the influence of the Coulomb scattering has had to be taken into account, too. Such inclusion of influence of the Coulomb interaction has been done, e.g., in the case of BSW model – see Ref. [41]. In the first four chapters of this paper it has been shown that this description cannot be done consistently by a simple substitution of these elastic hadronic amplitudes into the simplified complete WY amplitude (see Eqs. (1.3) and (1.5)). The other approach used in Ref. [47] has tried to incorporate the influence of the Coulomb scattering by means of calculating corrections to the relative WY phase factor. However, according to Section 3 such an approach is not sufficient or is not theoretically consistent.

However, it is possible to modify the given approaches to be in agreement with the eikonal model approach described in Section 3. The given procedure has been used by us for the first time in the case of model of Islam et al. (see Ref. [62]). Now it will be applied for the inclusion of influence of Coulomb interaction also in the case of the other three analyzed models.

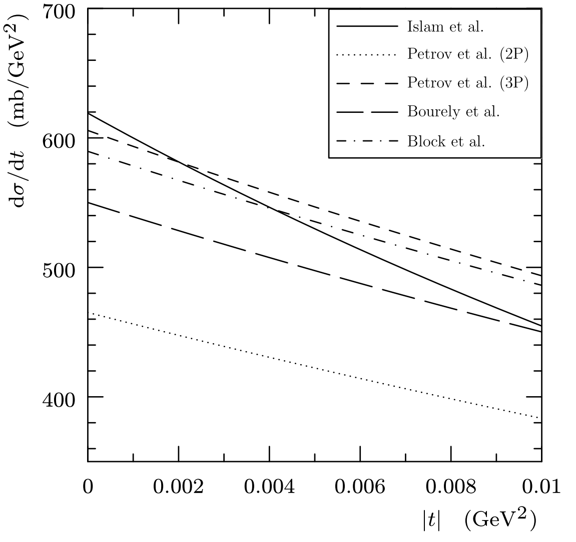

Fig. 4. $\frac{d\sigma}{dt}$ predictions of the hadronic scattering only at low $|t|$ for pp scattering at 14 TeV according to different models (in very forward direction).

Table 2
The values of basic parameters predicted by different models for pp elastic scattering at energy of 14 TeV.

Model	σ_{tot} [mb]	σ_{el} [mb]	$B(0)$ [GeV^{-2}]	$\rho(0)$
Bourrely et al.	103.64	28.51	20.19	0.121
Petrov et al. (2P)	94.97	23.94	19.34	0.097
Petrov et al. (3P)	108.22	29.70	20.53	0.111
Block et al.	106.74	30.66	19.35	0.114
Islam et al.	109.17	21.99	31.43	0.123

Each of the discussed phenomenological models has contained some number of free parameters in the formulas describing their s and t dependences. They have been determined by fitting their predictions on the corresponding experimental data at several lower energies of pp and $\bar{p}p$ elastic scattering. Their values will be taken from the quoted papers.

The predicted shapes of $\frac{d\sigma}{dt}$ for pp elastic hadronic scattering only at the nominal energy of 14 TeV are then shown in Fig. 4 at small $|t|$ region. Its different values in forward direction lead to different predicted values of the total cross section corresponding to each of the analyzed models; they have been determined with the help of the optical theorem – see Eq. (3.12). Their values can be found in Table 2. It contains also the values of the integrated elastic hadronic cross sections which have been determined by integration of modified Eq. (1.1) containing only hadronic amplitude $F^N(s,t)$.

The t dependences of predicted complete differential cross section values (i.e., including the influence of Coulomb scattering) are shown in Fig. 5. All the analyzed models show the diffractive minimum at $t \sim -0.4$ GeV^2. The values of $\frac{d\sigma}{dt}$ at the diffraction dip differ significantly (within one or two orders of magnitude). Some models have predicted also the appearance of diffractive structure at higher values of $|t|$. Let us point out especially the existence of the second diffractive dip being predicted by Bourrely, Soffer and Wu model [40].

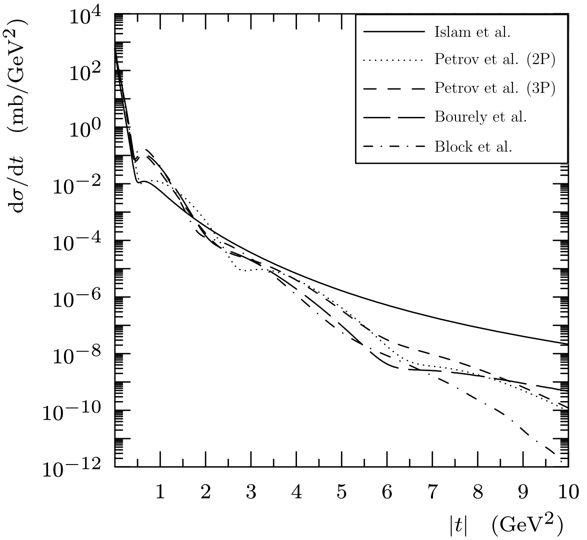

Fig. 5. $\frac{d\sigma}{dt}$ predictions for complete pp scattering at 14 TeV according to different models (in a larger interval of t).

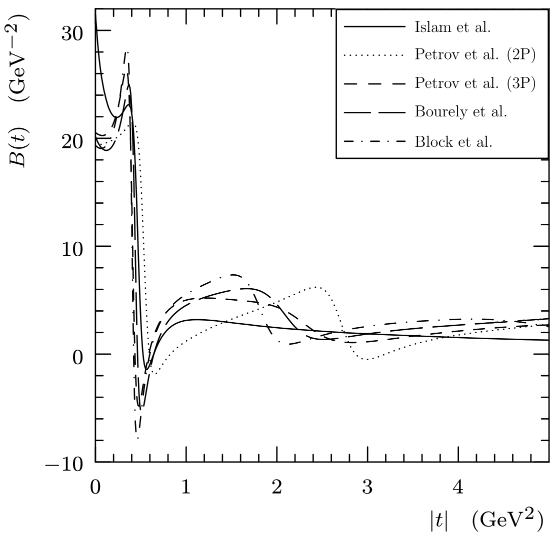

Fig. 6. The diffractive slope predictions for pp scattering at 14 TeV according to different models.

The predicted values of the diffractive slope $B(s,t)$ and of the quantity $\rho(s,t)$ have been determined with the help of formulas (3.10) and (3.11). The t dependences of the diffractive slope $B(t)$ are shown in Fig. 6. The model predictions differ at higher $|t|$ values; all of them are different from the constant dependence required in the simplified WY formula (1.3). Fig. 7 displays the t dependences of the quantity $\rho(t)$ which exhibit strong t dependence again in contradiction with the mentioned simplified WY complete amplitude. The predicted values of all these quantities are given in Table 2. They differ for divers models rather significantly; the total

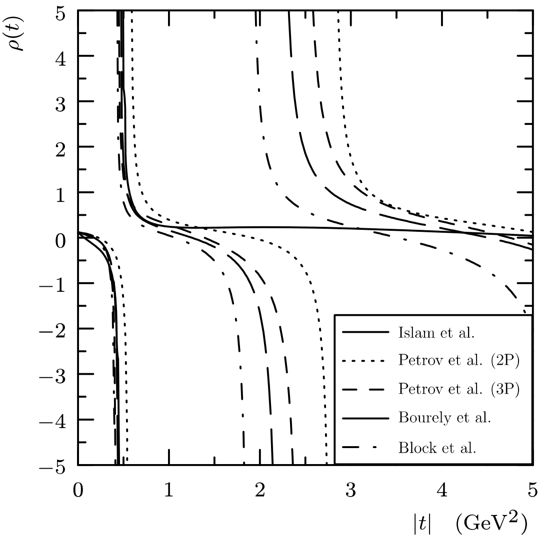

Fig. 7. The $\rho(t)$ predictions for pp scattering at 14 TeV according to different models.

cross section predictions range from 95 mb to 110 mb. Another value of 101.5 mb following from the formula

$$\sigma_{tot}(s) = 21.70\left(\frac{s}{s_0}\right)^{0.0808} + 56.08\left(\frac{s}{s_0}\right)^{-0.4525} \text{ [mb]}, \quad s_0 = 1\ \text{GeV}^2 \tag{5.1}$$

has been given by Donnachie and Landshoff [63] with the help of Regge pole model fit of pp total cross sections performed at lower energies. The higher value of $\sigma_{tot} = 111.5 \pm 1.2^{+4.1}_{-2.1}$ mb has been predicted by COMPETE Collaboration [64], having been determined by extrapolation of the fitted lower energy data with the help of dispersion relations technique. Let us remark that there is no reliable theoretical prediction for this quantity: e.g., the latest prediction on the basis of QCD for this quantity has been 125 ± 25 mb [65].

Fig. 8 represents then the t dependence of the ratio of interference to hadronic contributions of the $\frac{d\sigma}{dt}$ for all of the given models, i.e., the quantity

$$Z(t) = \frac{|F^{C+N}(s,t)|^2 - |F^C(s,t)|^2 - |F^N(s,t)|^2}{|F^N(s,t)|^2}. \tag{5.2}$$

The graphs show clearly that the influence of the Coulomb scattering may hardly be fully neglected at higher values of $|t|$. This fact can be considered as the next argument justifying the use of eikonal model approach instead of commonly used WY approach as it is commonly believed. It is interesting that at least for small $|t|$ the given characteristics are very similar for different models.

6. Luminosity estimation on the basis of pp elastic scattering at the LHC

Accurate determination of the elastic amplitude at a given energy is very important in the case when the luminosity of the collider is to be calibrated on the basis of elastic nucleon scattering. The luminosity $\mathcal{L}$ relates the experimental elastic differential counting rate $\frac{dN_{el}}{dt}(s,t)$ to the complete elastic amplitude $F^{C+N}(s,t)$ (see Eq. (1.1) and Refs. [7,8]) by

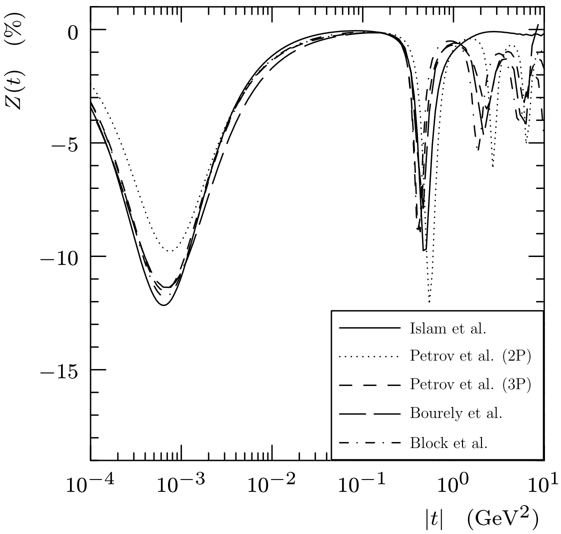

Fig. 8. The t dependence of the ratio of the interference to the hadronic contributions to the $\frac{d\sigma}{dt}$ for pp elastic scattering at 14 TeV according to different models.

$$\frac{1}{\mathcal{L}}\frac{dN_{el}}{dt}(s,t) = \frac{\pi}{sp^2}\left|F^{C+N}(s,t)\right|^2. \tag{6.1}$$

Eq. (6.1) is to be valid for any kinematically admissible value of t. The value $\mathcal{L}$ might be, however, calibrated in principle by measuring the counting rate in the region of the smallest $|t|$ where the Coulomb amplitude is dominant. This region may be, of course, hardly reached at the nominal LHC energy due to technical limitations. A procedure allowing to avoid these difficulties may be based on Eq. (6.1), when the elastic counting rate may be, in principle, measured at any t which can be reached, and the complete elastic scattering amplitude $F^{C+N}(s,t)$ may be determined with required accuracy at any $|t|$, too. However, in this case it will be very important which formula for the complete elastic amplitude $F^{C+N}(s,t)$ will be used.

We have studied the differences between the WY simplified formula (see Eqs. (1.3) and (1.5)) and the eikonal model (Eqs. (3.7)–(3.9)). The differences can be well visualized by the quantity

$$R(t) = \frac{|F^{C+N}_{eik}(s,t)|^2 - |F^{C+N}_{WY}(s,t)|^2}{|F^{C+N}_{eik}(s,t)|^2}, \tag{6.2}$$

where $F^{C+N}_{eik}(s,t)$ is the complete eikonal elastic scattering amplitude calculated for the studied hadronic model amplitude, and $F^{C+N}_{WY}(s,t)$ is the WY one (given by Eqs. (1.3) and (1.5)). The quantity $R(t)$ is plotted in Fig. 9 for several models.

The maximum deviations are expected to lie approximately in the center of interference domain, i.e., at

$$|t_{int}| \approx \frac{8\pi\alpha}{\sigma_{tot}} \approx 0.00064\ \text{GeV}^2 \tag{6.3}$$

where the Coulomb and the hadronic effects are expected to be practically equal [7,8]. Let us emphasize that the differences between the theoretically consistent eikonal model and the WY formula may reach here almost 5%. It means that the luminosity derived on the basis of

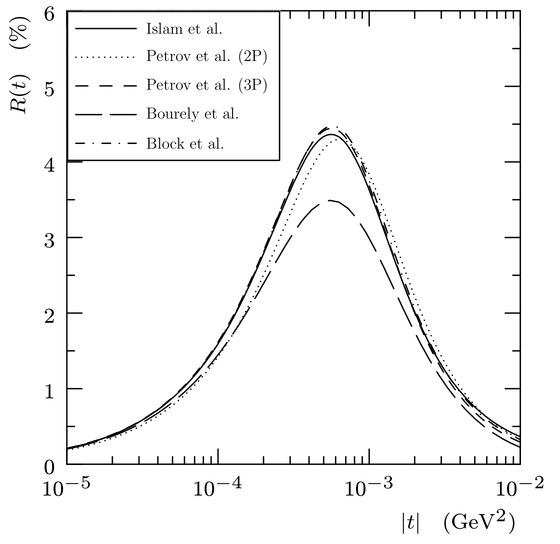

Fig. 9. The $R(t)$ quantity predictions for pp scattering at 14 TeV for different models from Section 6.

standard description of elastic pp scattering at the energy of 14 TeV might be burdened by a non-negligible systematic error, if determined only from a small t region around t_{int}. The given question should be, therefore, yet further analyzed.

It is, of course, possible to estimate also the ratio of real and imaginary parts of elastic hadronic amplitude, i.e., the quantity $\rho(t)$. This quantity is t dependent at difference to the standard description being based on simplified WY amplitude.

Minimal value $|t_{\min}|$ of momentum transfer which can be achieved in the elastic nucleon experiments is given by the detector acceptance which depends on the energy and on the value of betatron β^* function in the interaction point. In the TOTEM experiment at the energy of 14 TeV, $\beta^* = 1540$ m and 50% acceptance of detectors, $|t_{\min}|$ would be near approximately to 2×10^{-3} GeV^2. This value lies at the border on interference region; the deviation of luminosity estimation (according to different models, see Fig. 9) would be about 2% [69].

However, at lower energy of 7 TeV and the same $\beta^* = 1540$ m the value of $|t_{\min}|$ at 50% acceptance would be 8×10^{-4} GeV^2 which is nearly in the center of interference domain. The deviation of luminosity estimation would be about 4–5% if the standard WY approach would be used.

7. Root-mean-square values of impact parameter

Let us return yet to the problem of mean values of impact parameter for different kinds of collision processes that may be derived from experimental data with the help of eikonal model (see Section 3). The corresponding root-mean-square (RMS) values may be established for any elastic hadronic amplitude $F^N(s,t)$. It has been then possible to do it also in the case of models discussed in Section 5. Their values calculated with the help of formulas (3.23)–(3.25) are shown in Table 3. The values of elastic RMS are for all these models lower than the corresponding values of inelastic ones. It means that the elastic pp collisions would be much more central then the inelastic ones for all these models similarly as in the case of pp scattering at the ISR

Table 3
The values of root-mean-squares predicted by different models.

Model	$\sqrt{\langle b^2_{tot}\rangle}$ [fm]	$\sqrt{\langle b^2_{el}\rangle}$ [fm]	$\sqrt{\langle b^2_{inel}\rangle}$ [fm]
Bourrely et al.	1.249	0.876	1.399
Petrov et al. (2P)	1.227	0.875	1.324
Petrov et al. (3P)	1.263	0.901	1.375
Block et al.	1.223	0.883	1.336
Islam et al.	1.552	1.048	1.659

energies [66]. It is in disagreement with usual interpretation of a two matter objects collisions and it has been denoted, e.g., in Ref. [67] as a 'puzzle'. It is the consequence of admitting only a weak (standard) t dependence of elastic hadronic phase in all models. The given 'puzzle' can be easily removed if the elastic hadronic phase $\zeta^N(s,t)$ is allowed to have a more general shape of t dependence (see Refs. [15,70,71]).

This freedom in determination of phase $\zeta^N(s,t)$ is the consequence of the fact that two real functions (modulus and the phase) are to be derived from one experimentally established function (differential cross section). And it is the modulus $|F^N(s,t)|$ that is almost uniquely determined while the t dependence of the phase $\zeta^N(s,t)$ is only slightly limited. And it is possible to choose significantly different phase dependences [35]; see also Fig. 1.

And in standard approaches it is almost generally assumed that the imaginary part of elastic hadronic amplitude is dominant in a broad region of $|t|$ around the forward direction; being taken mostly as slowly decreasing with rising $|t|$ and vanishing at the diffractive minimum. The real part is assumed to start with a small value at $|t| = 0$, to become negative at small $|t|$ and to fill the dip in $d\sigma/dt$ in diffractive minimum. It means that the t dependence of phase $\zeta^N(s,t)$ is very weak and becomes significant only in the region of diffractive minimum; imaginary part of amplitude being put zero. However, *the existence of diffractive minimum does not require zero value for imaginary part at this point. Only the sum of both the squares of real and imaginary parts should be minimal at this point.* The mentioned assumption of vanishing imaginary part represents much stronger and more limiting condition than the physics requires.

Regarding Eq. (3.23) it is evident that very different elastic RMS values may be obtained according to the chosen t dependence of the phase $\zeta^N(s,t)$. One should distinguish between the so-called central picture (the first term dominates) and peripheral picture (decisive contribution comes from the second term when the phase increases quickly with rising t and reaches $\pi/2$ at $|t| \simeq 0.1$ GeV^2). The value $\langle b^2\rangle_{el}$ is lesser than $\langle b^2\rangle_{inel}$ in the central case while $\langle b^2\rangle_{el}$ is greater than $\langle b^2\rangle_{inel}$ in the peripheral case. In the central case the proton should be regarded as relatively transparent object which represents already puzzling question (see, e.g., Refs. [66,67]). And the more detailed analysis of models of elastic hadronic scattering giving the peripheral distribution of elastic hadronic scattering should be considered. No a priori limitations of elastic hadronic amplitude should be introduced in the corresponding analysis of experimental data.

The peripheral behavior seems to be slightly preferred on the basis of statistical analysis of pp experimental data at 53 GeV and $\bar{p}p$ at 541 GeV (see [15]). And the peripheral picture is supported also by analysis of elastic scattering of α particles on various targets (^{1}H, ^{2}H, ^{3}He, ^{4}He) performed [68] with the help of Glauber model where the 'elementary' nucleon–nucleon elastic hadronic amplitude has exhibited similar t dependence of phase $\zeta^N(s,t)$ as in our peripheral case – see Ref. [15].

8. Conclusion

In the past the analyses of high energy elastic nucleon scattering data in the region of very small $|t|$ were performed with the help of the simplified interference formula proposed by West and Yennie; including the influence of both Coulomb and hadronic interactions. At higher values of momentum transfers the influence of Coulomb scattering was then neglected and the elastic scattering of nucleons was described only with the help of hadronic amplitude having dominant imaginary part in a broad region of t and vanishing at the diffractive minimum. And it is evident that such a description of elastic nucleon scattering with the help of two different formulas for the complete elastic amplitude represents significant deficiency.

A more general eikonal model has been proposed. It describes elastic charged nucleon collisions at high energies by only one formula for the complete elastic amplitude in the whole kinematical region of t at a given energy value. This model is adequate for any t dependence of the elastic hadronic amplitude and has been successfully used for the analysis of elastic pp and $\bar{p}p$ scattering data at lower energies.

In several papers the attempts have been then done to look for some correlations between solutions found for individual energy values. We have used these models in combination with eikonal model to derive the predictions for some characteristics that might be established in the prepared experiment at the LHC energies [59,60]. At this place it is necessary to mention that some results contained in the presented in this paper have been included already in other papers; see also Refs. [70,71].

The attention has been also called to the problem of luminosity determination as the values of all other quantities are affected by its value. The model predictions indicate that a systematic difference up to 5% might occur between the eikonal and the West and Yennie formulas in the corresponding region of t values.

It is also necessary to call attention to the fact that the contribution of Coulomb scattering cannot be fully neglected at rather high $|t|$ values, either. However, the main open question concerns the fact that the experimental data of the differential cross section allow to derive directly the t dependence of the modulus, while the t dependence of the phase is only little constrained and may depend on some other assumptions or degrees of freedom. Any analysis of experimental data should, therefore, contain at the present always also statistical evaluation of two different alternatives: central and peripheral; peripheral behavior corresponding better to usual picture of collision processes. And more attention should be devoted to the construction of a model which would be able to represent a realistic picture of elastic hadronic scattering of charged nucleons.

Acknowledgement

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